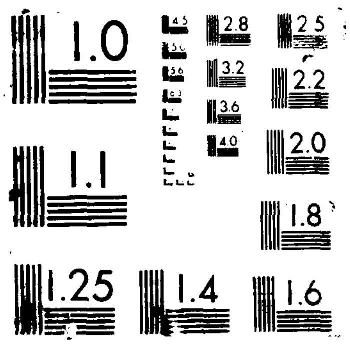
AD-A195 970 MATHEMATICAL PROBLEMS OF NONLINEAR WAVE PROPAGATION AND OF WAVES IN HETER..(U) STANFORD UNIV CA APPLIED MATHEMATICS GROUP J B KELLER 31 MAR 88 1/1

UNCLASSIFIED AFOSR-TR-88-0652 AFOSR-85-0087 F/G 20/4 NL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REPORT DOCUMENTATION PAGE

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1a. REPORT SECURITY CLASSIFICATION: Unclassified

1b. RESTRICTIVE MARKINGS:

2a. SECURITY CLASSIFICATION AUTHORITY:

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE:

3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; Distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S):

5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 88-0652

6a. NAME OF PERFORMING ORGANIZATION: Applied Mathematics Group, Stanford University

6b. OFFICE SYMBOL (If applicable):

7a. NAME OF MONITORING ORGANIZATION: AFOSR/NM

6c. ADDRESS (City, State and ZIP Code): Stanford California 94305

7b. ADDRESS (City, State and ZIP Code): Bld. 410, Bolling AFB, DC 20332

8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR

8b. OFFICE SYMBOL (If applicable): NM

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-85-0007B

8c. ADDRESS (City, State and ZIP Code): BLDG. 410, Bolling AFB, DC 20332

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO
61102F	2304	A4	

11. TITLE (Include Security Classification): Mathematical Problems of Nonlinear Wave Propagation and of Waves in Heterogeneous Media

12. PERSONAL AUTHOR(S): Professor Joseph B. Keller

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
FINAL	FROM 10/1/86 TO 9/30/87	March 31, 1988	8

16. SUPPLEMENTARY NOTATION:

17. COSATI CODES

FIELD	GROUP	SUB. GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number): nonlinear waves, heterogenous media, reciprocal theorems, effective parameters

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

In nonlinear wave propagation, Keller and Hunter have determined the asymptotic behavior of weakly nonlinear waves at caustics. They have also developed a theory of the propagation of short waves of any strength. Keller and Newton have found a method for analyzing the stability of a large class of nonlinear waves. Keller and Bonilla have shown how to deduce the theory of acoustoelasticity by considering nonlinear effects on waves in granular material.

In the theory of waves in heterogeneous media, Keller and Ahluwalia have analyzed scattering by slender bodies. Keller and Weinstein have determined the pass and stop bands for waves in stratified periodic media. Keller and Bai have done the same for an acoustic medium containing rigid spheres arranged in a simple cubic lattice. Keller has derived the amplitude equations for resonantly-interacting water waves in water of nonuniform depth.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION: Unclassified

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22c. OFFICE SYMBOL:

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AFOSR-TR- 88-0652

MATHEMATICAL PROBLEMS OF NONLINEAR WAVE PROPAGATION AND OF WAVES IN HETEROGENEOUS MEDIA

FINAL REPORT

October 1, 1986 - September 30, 1987

Professor Joseph B. Keller

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Grant No.: AFOSR-85-0007B

I. Brief Outline of Research Findings

The research findings under this grant are contained in the research papers listed in Section II of this report. Some of them have been published already, others have been submitted for publication and accepted, and others have not yet been accepted. The status of each paper is indicated after its title. In addition in Section III we give abstracts of papers submitted during this period which concern waves. Now we shall mention some the findings explicitly.

In nonlinear wave propagation, Profs. Keller and Hunter have determined the asymptotic behavior of weakly nonlinear waves at caustics. They have also developed a theory of the propagation of short waves of any strength. Profs. Keller and Newton have found a method for analyzing the stability of a large class of nonlinear waves. Profs. Keller and Bonilla have shown how to deduce the theory of acoustoelasticity by considering nonlinear effects on waves in granular material.

In the theory of waves in heterogeneous media, Profs. Keller and Ahluwalia have analyzed scattering by slender bodies. Profs. Keller and Weinstein have determined the pass and stop bands for waves in stratified periodic media. Profs. Keller and Bai have done the same for an acoustic medium containing rigid spheres arranged in a simple cubic lattice. Prof. Keller has derived the amplitude equations for resonantly-interacting water waves in water of nonuniform depth.

II. Manuscripts Published and Submitted for Publication

1. "Review of *Stochastic Wave Propagation*" by K. Sobcyzk", *SIAM Review* **28** 593–594 1986.
2. "Scattering by a slender body", (with D.S. Ahluwalia) *JASA* **80** 1782–1792 1986.
3. "Finite elastic deformation governed by linear equations", *J. Appl. Mech.* **53** 819–820 1986.
4. "Pouring Flows", (with J.M. Vanden-Broeck) *Phys .Fluids* **29** 3958–3961 1986.
5. "Finite amplitude vortices in curved channel flow", (with W.H. Finlay and J.H. Ferziger), Proc. of the 25th AIAA Aerospace Science Meeting, Reno, Jan. 1987.
6. "Free surface flow due to a sink", (with J.M. Vanden-Broeck) *J. Fluid Mech.* **175** 109–117 1987.
7. "Weir Flows", (with J.M. Vanden-Broeck) *J. Fluid Mech.* **176** 283–293 1987.
8. "Acoustoelasticity", *Dynamical problems in continuum physics*, eds. J.L. Bona, C. Dafermos, J.L. Ericksen and D. Kinderlehrer, Springer-Verlag, New York, 1987, pp. 193–203.
9. "Impact with an impulsive frictional moment", *ASME J. Appl. Mech.* **54** 239–240 1987.
10. "Effective conductivities of reciprocal media", *Random Media*, ed. G. Papanicolaou, Springer-Verlag, New York, 1987, pp. 183–188.
11. "Addendum: Asymptotic theory of nonlinear wave propagation", (with S. Kogelman), *SIAM J. Appl. Math.* **47** 454 1987.
12. "Caustics of nonlinear waves"(with J. K. Hunter), *Wave Motion* **9** 429–443 1987.
13. "Asymptotic behavior of stability regions for Hill's equation", (with Michael Weinstein), *SIAM J. Appl. Math.* **47** 941–958 1987.
14. "Stability of periodic plane waves," (with P.K.Newton) *SIAM J.Appl. Math.* **47** 959–964 1987.
15. "Sound waves in a medium containing rigid spheres," (with Dov Bai) *JASA* **82** 1436 1441 1987.
16. "Effective conductivity of periodic composites composed of two very unequal conductors," *J. Math. Phys.* **28** 2516–2520 1987.

17. "Lower bounds on permeability" (with Jacob Rubinstein) *Phys. Fluids* **30** 2919–2921 1987.

18. "Ropes in Equilibrium", (with J. Maddocks), *SIAM J. Appl. Math.* **47** 1185–1200 1987.

19. "Misuse of game theory," *J. Chronic Diseases*, **40** 1147–1148 1987.

20. "Stability of plane wave solutions of nonlinear systems" (with P.K. Newton), *Wave Motion*, in press.

21. "Fair Dice," *American Math. Monthly*, in press (June 1988).

22. "Precipitation Pattern Formation," (with M. Falkowitz), *J.Chem.Phys.*, **88** 416–421 1988.

23. "Resonantly interacting water waves," *J. Fluid Mech.*, in press.

24. "Nonlinear hyperbolic waves," (with J. K. Hunter), Proc. Royal Society, in press.

25. "Fast reaction, slow diffusion and curve shortening" (with J. Rubinstein and P. Sternberg), *SIAM J. Appl. Math.*, in press (12/87).

26. "Finite amplitude vortices in curved channel flow," (with W.H. Finlay and J.H. Ferziger), *J. Fluid Mech.*, in press.

27. "Newton's Second Law", *Am. J. Phys*, **55** 1145–1146 1987.

28. "Approximations for errors-in-variables regression," submitted 8/20/86, resubmitted 4/87, 11/17/87.

29. "Spilling" in *Science, Food and Drink* ed. Kuerti, submitted 6/5/87.

III. Abtstracts of Manuscripts Submitted During Report Period

Pouring flows, by J.M. Vanden-Broeck and J.B. Keller.

Free surface flows of a liquid poured from a container are calculated numerically for various configurations of the lip. The flow is assumed to be steady, two dimensional, and irrotational; the liquid is treated as inviscid and impressible; and gravity is taken into account. It is shown that there are jetlike flows with two free surfaces, and other flows with one free surface which follow along the underside of the lip or spout. The latter flows occur in the well-known "teapot effect," which was treated previously without including gravity. Some of the results are applicable also to flows over weirs and spillways.

Free surface flow due to a sink, by J.M. Vanden-Broeck and J.B. Keller.

Two-dimensional free surface flows without waves, produces by a submerged sink in a reservoir, are computed numerically for various configurations. For a sink above the horizontal bottom of a layer of fluid, there are solutions for all values of the Froude number F greater than some particular value. However, when the fluid is sufficiently deep, there is an additional solution for one special value of $F < 1$. The results for a sink at the vertex of a sloping bottom, treated by Tuck and Vanden-Broeck, are confirmed and extended. In particular it is shown that as the bottom tends to the horizontal, the solution for a sink at the vertex of a sloping bottom approaches a solution for a horizontal bottom with $F = 1$. However solutions are found for all values of the Froude number $F \geq 1$ for a sink on a horizontal bottom.

Weir flows, by J.M. Vanden-Broeck and J.B. Keller.

The flow of a liquid with a free surface over a weir in a channel is calculated numerically for thin weirs in channels of various depths, and for broad-crested weirs in channels of infinite depth. The results show that the upstream velocity, as well as the entire flow, are determined by the height of the free surface far upstream and by the geometry of the weir and channel, in agreement with observation. The discharge coefficient is computed for a thin weir, and a formula for it is given that applies when the height of the weir is large compared to the height of the upstream free surface above the top of the weir. The coefficients in this formula are close to those found empirically.

Caustics of nonlinear waves, by J.K. Hunter and J.B. Keller.

The behavior at caustics is analyzed for weakly nonlinear wave solutions of hyperbolic equations. It is shown that short waves, weak enough to be governed by linear or weakly nonlinear geometrical optics away from caustics, are governed by linear theory at and near caustics. For somewhat stronger waves, for which linear theory does not suffice at caustics, a weakly nonlinear caustic theory is developed. It leads to an equation derived by Guiraud, Hayes, and Seebass for gas dynamics.

Asymptotic behavior of stability regions for Hill's equation, by M. Weinstein and J.B. Keller.

The asymptotic behavior of the solutions of Hill's equation $u'' + [E - \lambda^2 q(x)]u = 0$ is determined for large positive real values of the coupling constant λ^2 and large real values of the energy E. The locations and widths of the stability bands and instability gaps are found. The band widths are shown to decrease exponentially as λ increases when $\lambda^{-2}E$ lies between the minimum and maximum values of the periodic potential $q(x)$. The gap widths decrease exponentially with λ when $\lambda^{-2}E$ is greater than the maximum of $q(x)$. For $\lambda^{-2}E$ asymptotically equal to the maximum of $q(x)$, the width of the nth band is asymptotically half the width of the nth gap. The exponentially small band and gap widths are related to the exponentially small transmission and reflection coefficients, associated with one period of $q(x)$. The present results extend previous ones of Meixner and Schäfke, Harrell, and the authors, in which $\lambda^{-2}E$ was near the minimum of $q(x)$.

Stability of periodic plane waves, by P.K. Newton and J.B. Keller.

Nonlinear partial differential equations with explicit periodic plane wave solutions are considered. For a class of such equations the variational equations governing linear stability are solved explicitly also. An explicit equation for the growth rate of a perturbation is obtained. The analysis is applied to nonlinear forms of the Schrödinger and Klein-Gordon equations and to a generalized Korteweg-de Vries equation.

Sound waves in a periodic medium containing rigid spheres, by D. Bai and J.B. Keller.

The effective speed of sound is calculated for a periodic composite

medium consisting of a gas or liquid containing immovable rigid spheres arranged in a simple cubic lattice. Long waves propagating along a lattice axis are treated. For such waves, the wave equation for the pressure can be reduced to the Webster horn equation. This is an ordinary differential equation to which Floquet theory is applied. Both perturbation and numerical methods are used to find the effective speed of sound as a function of frequency, and to locate the passbands and the stop bands.

Lower bounds on permeability, by J. Rubinstein and J.B. Keller.

A method is presented for obtaining lower bounds on the permeability of a porous medium. It is applied to media composed of periodic and random configurations of spheres and cylinders.

Newton's Second Law, by J.B. Keller.

Newton's second law is interpreted in terms of the matrix of accelerations of different bodies produced by different motion-altering agents. The law asserts that such a matrix can be factored into a row matrix times a column matrix. This factorization leads to a definition of mass, a definition of force, and an expression for acceleration in terms of mass and force.

Stability of plane wave solutions of nonlinear systems, by P.K. Newton and J.B. Keller.

The linear stability of plane wave solutions of various systems of nonlinear partial differential equations is treated. They include an equation for the envelope of a surface wave train on deep water, Zakharov's system for Langmuir waves in plasmas, coupled Schrödinger and Klein-Gordon equations for nucleon and meson fields, and a pair of coupled Schrödinger equations. The method of analysis, which was presented previously by the authors, is explained and its scope is broadened.

Resonantly interacting water waves, by J.B. Keller.

Coupled nonlinear equations are derived for the amplitudes of two small amplitude resonantly-interacting gravity waves in water of nonuniform depth. Such resonance is possible only for wavelengths long compared to the depth. It is shown that the same equations are obtained from the exact Euler equations, from the Nonlinear Shallow Water theory, and from the Boussinesq equations.

Nonlinear hyperbolic waves, by J.K. Hunter and J.B. Keller.

A theory describing the propagation of nonlinear hyperbolic waves of any strength is developed. It is valid for small values of the wavelength, i.e. of the typical scale length of variation in the direction of propagation. At first the wave propagates along wave normals according to one dimensional theory. It quickly splits up into a set of distinct waves, each of which soon becomes weak. The weak waves then propagate along rays according to weakly nonlinear geometrical optics.

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